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REGIONAL ANALYSIS OF MAXIMUM RAINFALL USING L-MOMENT AND LQ-MOMENT: A COMPARATIVE CASE STUDY FOR THE NORTH EAST INDIA

DHRUBA JYOTI BORA[1], MUNINDRA BORAH[2] & ABHIJIT BHUYAN[3]

[1,2]Department of Mathematical Sciences, Tezpur University, Napaam, Tezpur, Assam, India

[3]Department of Mathematics, C.K.B College, Teok, Assam, India

ABSTRACT

Rainfall data of twelve gauged stations of the North East India has been taken for selecting best fit model for rainfall frequency analysis. The five probability distributions, namely Generalized extreme value (GEV), Generalized Logistic (GLO), Pearson type 3 (PE3), 3 parameter Log normal (LN3) and Generalized Pareto (GPA) distributions have been considered. The methods of L-moment and LQ-moment have been used for estimating the parameters of the probability distributions. L-moment analysis shows that PE3 is the best fitting distribution. On the other hand based on LQ-moment analysis GPA is selected as the best fitting distribution for the North Eastern Region. Relative root mean square error (RRMSE) and RBIAS are employed to compare between the results found from L-moment and LQ-moment analysis. It is found that PE3 distribution is selected as suitable and the best fitting distribution for rainfall frequency analysis of the North East India. Also the L-moment method is significantly more efficient than LQ-moment for rainfall frequency analysis of the North east India. The rainfall frequency model for the region has been developed by using the identified robust distribution for the region.

KEYWORDS: L-Moments, LQ-Moments, Probability Distribution, RRMSE Error

1. INTRODUCTION

Agriculture plays a vital role for the economic development of the North East India. Rainfall has direct impact on agriculture. Therefore, the rainfall frequency analysis of this region is one of the essential tasks. Also the study of extreme rainfall is very much useful for design of dam, bridge and hydrological planning. The proper estimation of rainfall frequency analysis will help to develop and also protect the economic loss of this region.

In this study regional rainfall frequency analysis of North East India has been considered for development of frequency analysis model. For this study the L-moment and LQ-moment method has been used for estimation of parameters of the probability distributions. The five probability distributions, namely generalized extreme value (GEV), generalized Logistic(GL), Pearson type 3 (PE3), 3 parameter Log normal (LN3) and generalized Pareto (GPA) distributions have been considered for this study. The homogeneity of the study region has been carried out by using heterogeneity measure proposed by Hosking and Wallis (1993). Two goodness of fitness measures namely Z-statistics and L-moment (LQ-moment) ratio diagram have been employed for identification of the best fitting distribution for our study region. Also RRMSE and RBIAS is used to make a comparison between the two best fitting distribution getting from L-moment and LQ-moment analysis.

2. REVIEW OF LITERATURE

Application of extreme value distribution to rainfall data have been investigated by several authors from different parts of the world. Shabri, A. B. et. al (2011) used L-moment and TL-moment to analysis the maximum rainfall data of 40 stations of Selangor Malaysia. Comparison between the two approaches showed that the L-moments and TL-moments produced equivalent results. GLO and GEV distributions were identified as the most suitable distributions for representing the statistical properties of extreme rainfall in Selangor. Monte Carlo simulation was used for performance evaluation, and it showed that the method of TL-moments was more efficient for lower quantile estimation compared with the L-moments. Deka, S. et. al (2011) fitted three extreme value distributions using LH moment of order zero to four and found that GPA distribution is the best fitting distribution for the majority of the stations in North East Region of India. Also Deka, S. et. al (2009) tried to determine the best fitting distribution to describe the annual series of maximum daily rainfall data for a period of 42 years of nine stations of North East Region of India. Generalised Logistic distribution is empirically proved to be the most appropriate distribution for the majority of the stations in North East Region of India. Olofintoye, O.O. et. al (2009) analysed annual rainfall data of 54 years from 20 different stations of Nigeria using Gumbel, Log-Gumbel, Normal, Log-Normal, Pearson and Log-Pearson distribution. The result showed that the Log-Pearson Type III distribution was the best distribution. Lee (2005) studied the rainfall distribution characteristics of Chia-Non plain area by using different statistical analyses such as normal distribution, log-normal distribution, extreme value type I distribution, Pearson type III distribution and log-Pearson type III distribution. The result showed that the log-Pearson type III distribution performed the best probability distribution occupying 50% of the total station number. Zalina et. al (2002) studied maximum rainfall frequency analysis of Malaysia and found that GEV distribution is the most appropriate distribution for describing the annual maximum rainfall series in Malaysia. Ogunlela (2001) studied the stochastic analysis of rainfall event in Ilorin using probability distribution functions. He found that the log Pearson type III distribution is the best for describing peak daily rainfall data of Ilorin.

3. STUDY REGION AND DATA COLLECTION

For this study 12 gauged stations of the North East India viz. Imphal, Agartala, Shillong, Guwahati, Silchar, Jorhat, Dhubri, Lengpui, Lakhimpur, Pasighat, Mohanbari and Itanagar are considered. Annual daily maximum rainfall data of these stations for a period of 30 years from 1984 to 2013 are considered for this study. Data are collected from Regional Meteorological centre, Guwahati.

4. METHODOLOGY

4.1 Method of L-Moment

The probability weighted moments (PWMs) of a random variable X with cumulative distribution function (CDF), $F(.)$ were defined by Greenwood et al. (1979) as

$$\beta_r = M_{1,r,0} = E[X\{F(X)\}^r] \tag{4.1.1}$$

where, $$M_{p,r,s} = E[X^p\{F(X)\}^r\{1-F(X)\}^s] \tag{4.1.2}$$

β_r and can be rewritten as:

$$\beta_r = \int_0^1 x(F)F^r dF \,, \qquad r = 0,1,2 \ldots . \tag{4.1.3}$$

where $x(F)$ is the inverse CDF of x evaluated at the probability F.

The general form of L-moments in terms of PWMs is given by Hosking and Wallis (1997) as

$$\lambda_{r+1} = \sum_{k=0}^{r} p^*_{r,k}\beta_k \tag{4.1.4}$$

where, $p^*_{r,k}$ defined by Hosking and Wallis (1997) as

$$p^*_{r,k} = \frac{(-1)^{r-k}(r+k)!}{(k!)^2(r-k)!} \tag{4.1.5}$$

The first four L-moments can be defined as:

$$\lambda_1 = \beta_0 \tag{4.1.6}$$

$$\lambda_2 = 2\beta_1 - \beta_0 \tag{4.1.7}$$

$$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0 \tag{4.1.8}$$

$$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0 \tag{4.1.9}$$

Hosking and Wallis (1997) defined L-moments ratios (LMRs) as:

Coefficient of L-variation, $\tau = \lambda_2/\lambda_1$

Coefficient of L-skewness $\tau_3 = \lambda_3/\lambda_2$ (4.1.10)

Coefficient of L-kurtosis $\tau_4 = \lambda_4/\lambda_2$

4.2 Method of LQ-Moment

Let $X_1, X_2, X_3, \ldots\ldots X_n$ be a sample from a continuous distribution function $F_x(.)$ with quantile function $Q_x(u) = F_x^{-1}(u)$. If $X_{1:n} \leq X_{2:n} \leq X_{3:n} \leq \cdots \ldots \leq X_n$ denote the order statistics, then the rth LQ-moments $\zeta_r$of X proposed by Mudholkar. et al. (1998) are given by

$$\zeta_r = r^{-1}\sum_{k=0}^{k=r-1}(-1)^k\binom{r-1}{k}\tau_{p,\alpha(X_{r-k:r})}, \qquad r=1,2,\ldots. \tag{4.2.1}$$

where $0 \leq \alpha \leq \frac{1}{2}$, $0 \leq p \leq \frac{1}{2}$, and

$$\tau_{p,\alpha}(X_{r-k:r}) = pQ_{X_{r-k:r}}(\alpha) + (1-2p)Q_{X_{r-k:r}}(\alpha) + pQ_{X_{r-k:r}}(1-\alpha) \tag{4.2.2}$$

The linear combination $\tau_{p,\alpha}$ is a quick measure of the location of the sampling distribution of order statistic $X_{r-k:r}$. With appropriate combinations of $\alpha$and p, estimators for $\tau_{p,\alpha}(.)$ can be found which are functions of commonly used estimators such as median , trimean and Gastwirth. The trimean-based estimator is defined as

$$\frac{Q_{X_{r-k:r}}\left(\frac{1}{4}\right)}{4} + \frac{Q_{X_{r-k:r}}\left(\frac{1}{2}\right)}{2} + \frac{Q_{X_{r-k:r}}\left(\frac{3}{4}\right)}{4}$$

The first four LQ-moments of the random variable X are given by:

$$\zeta_1 = \tau_{p,\alpha}(X), \tag{4.2.3}$$

$$\zeta_2 = \frac{1}{2}[\tau_{p,\alpha}(X_{2:2}) - \tau_{p,\alpha}(X_{1:2})], \tag{4.2.4}$$

$$\zeta_3 = \frac{1}{3}[\tau_{p,\alpha}(X_{3:3}) - 2\tau_{p,\alpha}(X_{2:3}) + \tau_{p,\alpha}(X_{1:3})], \tag{4.2.5}$$

$$\zeta_4 = \frac{1}{4}[\tau_{p,\alpha}(X_{4:4}) - 3\tau_{p,\alpha}(X_{3:4}) + 3\tau_{p,\alpha}(X_{2:4}) - \tau_{p,\alpha}(X_{1:4})] \tag{4.2.6}$$

The LQ-CV, LQ-skewness and LQ-kurtosis are defined by

$$\eta = \frac{\zeta_2}{\zeta_1}, \eta_3 = \frac{\zeta_3}{\zeta_2} \text{ and } \eta_4 = \frac{\zeta_4}{\zeta_2} \tag{4.2.7}$$

5. REGIONAL RAINFALL FREQUENCY ANALYSIS USING L-MOMENT

5.1 Screening of Data

The Discordancy test, D_i, proposed by Hosking and Wallis (1993) has been used to screen out data from stations whose point sample L-moments are markedly different from other stations. The objective is to check the

$$D_i = \frac{1}{3}N(u_i - \bar{u})^T S^{-1}(u_i - \bar{u}) \tag{5.1.1}$$

where $S = \sum_{i=1}^{N}(u_i - \bar{u})(u_i - \bar{u})^T$ and $u_i = \left[t_2^i, t_3^i, t_4^i\right]^T$ for i-th station, N is the number of stations , S is covariance matrix of u_i and $\bar{u}$ is the mean of vector, u_i . Critical values of discordancy statistics are tabulated by Hosking and Wallis (1993), for $N = 12$, the critical value is 2.757. If the D-statistics of a station exceeds 2.757, its data is discordant from the rest of the regional data.

5.2 Heterogeneity Measure

An essential task in regional frequency analysis is the determination of homogeneous regions. Hosking and Wallis (1993) suggested the heterogeneity test, H, where L- moments are used to assess whether a group of stations may reasonably be treated as belonging to a homogeneous region. The proposed heterogeneity tests are based on: the L-co-efficient of variation (L-Cv), L-skewness (L-Sk) and L-kurtosis (L-Ck). These tests are defined respectively as

$$\mathrm{V}_1 = \sqrt{\textstyle\sum_{i=1}^{N} n_i (t_2^{(i)} - t_2^R)^2 / \sum_1^N n_i} \tag{5.2.1}$$

$$\mathrm{V}_2 = \textstyle\sum_{i=1}^{N}\{n_i[(t_2^{(i)} - t_2^R)^2 + \left(t_3^{(i)} - t_3^R\right)^2]^{\frac{1}{2}}\} / \sum_{i=1}^{N} n_i \tag{5.2.2}$$

$$\mathrm{V}_3 = \textstyle\sum_{i=1}^{N}\{n_i[(t_3^{(i)} - t_3^R)^2 + \left(t_4^{(i)} - t_4^R\right)^2]^{\frac{1}{2}}\} / \sum_{i=1}^{N} n_i \tag{5.2.3}$$

The regional average L-moment ratios are calculated using the following formula

$$\begin{aligned} t_2^R &= \textstyle\sum_{i=1}^{N} n_i\, t_2^i / \sum_{i=1}^{N} n_i, \\ t_3^R &= \textstyle\sum_{i=1}^{N} n_i\, t_3^i / \sum_1^N n_i, \\ t_4^R &= \textstyle\sum_{i=1}^{N} n_i\, t_4^i / \sum_1^N n_i \end{aligned} \tag{5.2.4}$$

where N is the number of stations and n_i is the record length at i-th station. The heterogeneity test is then defined as

$$\mathrm{H_j} = \frac{\mathrm{V_j} - \mu_{V_j}}{\sigma_{\mathrm{V_j}}}, \quad \mathrm{j} = 1,2,3 \tag{5.2.5}$$

where $\mu_{\mathrm{V_j}}$ and $\sigma_{\mathrm{V_j}}$ are the mean and standard deviation of simulated V_j values, respectively. The region is acceptably homogeneous, possibly homogeneous and definitely heterogeneous with a corresponding order of L-moments according as H<1, 1≤H<2 and H≥2.

The heterogeneity measures of our study region have been found to be $H_1 = 1.54, \mathrm{H}_2 = -0.35$and $\mathrm{H}_3 = 0.40$. It has been observed from heterogeneity measures that, our study region can be considered as a possibly homogeneous one.

5.3 Goodness of Fit Measures

5.3.1 Z-Statistics Criteria

The Z-test judges how well the simulated L-Skewness and L-kurtosis of a fitted distribution matches the regional average L-skewness and L-kurtosis values. For each selected distribution, the Z-test is calculated as follows

$$\mathrm{Z}^{\mathrm{DIST}} = (\tau_4^{\mathrm{Dist}} - \mathrm{t}_4^{\mathrm{R}})/\sigma_4 \tag{5.3.1}$$

where DIST refers to a particular distribution, τ_4^{DIST} is the L-kurtosis of the fitted distribution while the standard deviation of $\mathrm{t}_4^{\mathrm{R}}$ is given by

$$\sigma_4 = \left[(N_{sim})^{-1} \sum_{m=1}^{N_{sim}} (t_4^{(m)} - t_4^R)^2\right]^{1/2} \tag{5.3.2}$$

$\mathrm{t}_4^{\mathrm{m}}$is the average regional L-kurtosis and has to be calculated for the m^{th} simulated region. This is obtained by simulating a large number of kappa distribution using Monte Carlo simulations. The value of the Z-statistics is considered to be acceptable at the 90% confidence level if $|\mathrm{Z}^{\mathrm{DIST}}| \leq 1.64$. If more than one candidate distribution is acceptable, the one with the lowest $|Z^{DIST}|$ is regarded as the best fit distribution.

The Z-statistics values of five distribution used for our study are given in Table-2. It has been observed that the Z-statistic values of GEV, LN3 and PE3 distributions are less than 1.64. But that of PE3 distribution is the lowest i.e. 0.19. Therefore the PE3 distribution is identified as the best fitting distribution for rainfall frequency analysis of North-East India.

5.3.2 L-Moment Ratio Diagram

L-moment ratio diagram is a graphical tool which can be used as another goodness of fit measure for selection of best fit distribution. It is a graph of the L-skewness and L-kurtosis which compares the fit of several distributions on the same graph. According to Hosking and Wallis (1997), the expression of τ_4 in terms of τ_3 for an assumed distribution is given by

$$\tau_4 = \sum_{k=0}^{8} A_k \tau_3^k \tag{5.3.2}$$

where the coefficients A_k are tabulated by Hosking and Wallis (1997).

The L-moment ratio diagram of our study region is shown in Figure 1. It has been observed from Figure 1 that the regional average values of L-skewness and L-kurtosis is also lies near to the PE3 distribution. Hence, the L-moment ratio diagram also shows that the PE3 distribution is the best fit distribution to our study area.

5.4 Quantile Estimation

The quantile function of the best fitting distribution PE3 is given by

$$Q(F) = \mu + \sigma Q_0(F) \tag{5.4.1}$$

where $Q_0(F) = \frac{2}{\gamma}\left[1 + \frac{\gamma\emptyset^{-1}(F)}{6} - \frac{\gamma^2}{36}\right]^3 - \frac{2}{\gamma}$ and $\emptyset^{-1}(.)$ has a standard normal distribution with zero mean and unit variance. Parameters γ, μ and σ are the standard parameterizations which can be obtained by setting

$\alpha = \frac{4}{\gamma^2}, \beta = \frac{1}{2}\sigma|\gamma|$ and $\xi = \mu - \frac{2\sigma}{\gamma}$

The regional parameters for the PE3 distributions are presented in Table 4. Substituting the regional values of PE3 distribution in equation (5.4.1) quantiles are estimated. The estimated quantiles are given in table 5.

6. REGIONAL RAINFALL FREQUENCY ANALYSIS USING LQ-MOMENT

6.1 Screening of Data

The procedure discussed in section 5.1 is employed for LQ-moment also. For discordancy test L-cv, L-skewness and L-kurtosis are replaced by LQ-cv, LQ-skewness and LQ-kurtosis respectively.

6.2 Heterogeneity Measure

The procedure discussed in section 5.2 is similarly employed for LQ-momenmt. For Heterogeneity test L-cv, L-skewness and L-kurtosis are replaced by LQ-cv, LQ-skewness and LQ-kurtosis respectively.

The heteroginity measures of our study region have been found to be $H_1 = -1.45, \mathrm{H}_2 = 0.87$and $\mathrm{H}_3 = 1.77$. It has been observed from heterogeneity measures that, our study region can be considered as a possibly homogeneous one.

6.3 Goodness of Fit Measures

6.3.1 Z-Statistics Criteria

The procedure is similar as discussed in section 5.3. For Z- test L-cv, L-skewness and L-kurtosis are replaced by LQ-cv, LQ-skewness and LQ-kurtosis respectively. The Z-statistics values of five distribution used for our study are given in Table-6 It has been observed that the Z-statistic values of GEV, LN3, PE3 and GPA distributions are less than 1.64. But that of GPA distribution is the lowest i.e. 0.46.Therefore the GPA distribution is identified as the best fitting distribution for rainfall frequency analysis of North-East India.

6.3.2 LQ-Moment Ratio Diagram

The procedure is similar as discussed in section 5.3.2. The coefficients $A_k$are calculated by Bhuyan A. & M. Borah. (2011). The LQ-moment ratio diagram of our study region is shown in Figure 2. It has been observed from Figure 2 that the regional average values of LQ-skewness and LQ-kurtosis is also lies near to the GPA distribution. Hence, the LQ-moment ratio diagram also shows that the GPA distribution is the best fit distribution to our study area.

6.4 Quantile Estimation

The quantile function of the best fitting distribution GPA is given by

$$\mathrm{Q(F)} = [\xi + \frac{\alpha}{k}\{1 - (1 - \mathrm{F})^{k}\}] \tag{6.4.1}$$

where Q(F) is the quantile estimate at return period F. ξ, α, k are the parameters. The parameters of the GPA distribution are given in table-9.

Substituting the parameters in equation (6.4.1) the quantiles are estimated. The estimated quantiles are given in table 10.

7. RESULTS AND DISCUSSION

For both L-moment and LQ-moment methods it is observed from table-1 and table-5 that the D_i values of all the twelve stations are less than critical value 2.757. Therefore all the data of twelve stations are considered for the development of regional frequency analysis.

It has been observed from heterogeneity measures that for both L-moment and LQ-moment methods, our study region can be considered as a possibly homogeneous one.

Z-statistics criteria for L-moment and L-moment ratio diagram shows that the PE3 distribution is the best fitting distribution for our study region. The parameters of PE3 distribution using L-moment and quantile estimates using PE3 distribution are given in table-3 and table-4 respectively.

On the other hand, Z-statistics criteria for LQ-moment and LQ-moment ratio diagram shows that GPA distribution is the best fitting distribution for our study region. The parameters of GPA distribution using LQ-moment and quantile estimates using GPA distribution are given in table-7 and table-8 respectively.

The robustness of the two best fitting distributions designated by L-moment and LQ-moment are also investigated using Monte Curlo simulation proposed by Meshgi and Khalili (2009). Commonly used two error functions are relative root mean square error (RRMSE) and relative bias (RBIAS) are given by

$$\mathrm{R}RMSE = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left(\frac{Q_T^m - Q_T^c}{Q_T^c}\right)^2}$$

$$RBIAS = \frac{1}{M}\sum_{m=1}^{M}\left(\frac{Q_T^m - Q_T^c}{Q_T^c}\right)$$

where M is the total number of samples, $Q_T^m$and Q_T^c are the simulated quantiles of mth sample and calculated quantiles from observed data respectively. The minimum RRMSE and RBIAS values and their associated variability are used to select the most suitable probability distribution function. For this purpose, box plots, a graphical tool introduced by Tukey (1977) are used.

Box plot is a simple plot of five quantities, namely, the minimum value, the 1stquantile, the median, the 3rdquantile, and maximum value. This provides the location of the median and associated dispersion of the data at specific probability levels. The probability distribution with the minimum achieved median RRMSE or RBIAS values, as well as the minimum dispersion in the median RRMSE or RBIAS values, indicated by both ends of the box plot are selected as the suitable distribution.

RRMSE and RBIAS values are given in table-9 and table-10 respectively. From table-9 it is observed that the RRMSE values of PE3 distribution are less than or equal to the respective RRMSE values of GPA distribution. Also from table-10 it is observed that the absolute RBIAS values of PE3 distribution are smaller than the respective RBIAS values of GPA distribution. Figure 3 and Figure 4 represent the box plot of RRMSE and RBIAS values respectively. From figure 3 and Figure 4 it is observed that PE3 distribution has the minimum median RRMSE and RBIAS values as well as minimum dispersion. Hence PE3 distribution is selected as suitable and the best fitting distribution for rainfall frequency analysis of the North East India. Also the L-moment method is significantly more efficient than LQ-moment for rainfall frequency analysis of the North east India.

8. DEVELOPMENT OF MODEL

The regional rainfall frequency relationship is developed by using suitable and the best fitting distribution PE3. The form of regional frequency relationship or growth factor for PE3 distribution is

$$Q(F) = \left[\mu + \sigma \frac{2}{\gamma}\left\{\left\{1 + \frac{\gamma\emptyset^{-1}(F)}{6} - \frac{\gamma^2}{36}\right\}\right\}^3 - \frac{2}{\gamma}\right] * \bar{Q} \quad (8.1)$$

where Q(F) is the quantile estimation at non-exeedance probability F, $\overline{Q}$ is the mean annual maximum rainfall of the site,$\emptyset^{-1}(.)$ has a standard normal distribution with zero mean and unit variance. Parameters γ, μ and σ are the standard parameterizations which are given in the table-4. Substituting these values in expression (8.1) rainfall frequency relationship for gauged sites of study area is expressed as:

$$Q(F) = \left[1.000 + \frac{0.604}{1.155}\left\{\left\{1 + \frac{1.155\emptyset^{-1}(F)}{6} - \frac{1.334}{36}\right\}\right\}^3 - \frac{2}{1.155}\right] * \bar{Q} \quad (8.2)$$

For estimation of rainfall of desired non-exceedance probability for a small to moderate size gauged catchments of study area, above regional flood frequency relationship may be used. Alternatively, rainfalls of various non-exceedance probabilities may also be computed by multiplying the mean annual rainfall of a gauge station by corresponding values of growth factors based on the PE3 distribution given in table-4.

9. CONCLUSIONS

For both the methods, L-moment and LQ-moment Discordancy measure shows that data of all gauging sites of our study area are suitable for using regional frequency analysis. By using the L-moment and LQ-moment based homogeneity test, the region has been found to be possibly homogeneous. Regional rainfall frequency analysis was performed using five frequency distributions: viz. GLO, GEV, GPA, LN3 and PE3. Using L-moment ratio diagram and Z-statistic it is found that PE3 distribution is the best fitting distribution for rainfall frequency analysis of the North East India. Also using LQ-moment ratio diagram and Z-statistic it is found that GPA is the best fitting distribution for rainfall frequency analysis of the North East India. Using RRMSE and RBIAS values it can be concluded that PE3 distribution for L-moment is the most suitable distribution for rainfall frequency analysis of the North East India. Also the L-moment method is significantly more efficient than LQ-moment for rainfall frequency analysis of the North east India. The regional flood frequency relationship for gauged stations has been developed for the region and can be used for estimation of rainfalls of desired return periods.

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APPENDIES

Table 1: Discordancy Measures of each Sites of the NE Region Using L-Moment

No. of Site	No. of Observation	Name of Sites	L-CV	L-Skewness	L-Kurtosis	D_i
1	30	Guwahati	0.1509	0.2298	0.1551	0.27
2	30	Mohanbari	0.1458	0.1521	.1011	0.09
3	28	Silchar	0.1449	0.1420	0.1345	0.61
4	30	Lakhimpur	0.1394	0.2107	0.0976	0.93
5	30	Passighat	0.2224	0.3773	0.3455	1.82
6	30	Agartala	0.1763	0.1421	0.0529	1.30
7	30	Imphal	0.1772	0.2015	0.1711	0.19
8	30	Shillong	0.1863	0.1569	0.1778	1.32
9	26	Itanagar	0.1710	0.3629	0.2452	1.45
10	22	Dhubri	0.1798	0.1620	0.0904	0.75
11	25	Jorhat	0.1196	-0.0514	-0.0832	1.72
12	13	Lengpui	0.1166	0.1523	0.1265	1.56

Table 2: Z-Statistics Values Of the Distributions Using L-Moment

Sl.No.	Name of the Probability Distribution	Z-Statistics Values
1	GLO	2.58
2	GEV	0.87
3	LN3	0.55
4	PE3	0.19
5	GPA	2.97

Table 3: Parameters of Best Fitting Distribution Using L-Moment

Name of Distribution	Parameters		
	Location(μ)	Scale(δ)	Shape(γ)
PE3	1.000	0.302	1.155

Table 4: Quantile Estimates by Using Best Fitting Distribution

Distribution	Quantiles									
	0.010	0.020	0.050	0.100	0.200	0.500	0.900	0.950	0.990	0.999
PE3	0.553	0.576	0.620	0.669	0.745	0.943	1.405	1.574	1.942	2.434

Table 5: Discordancy Measures of Each Sites of the NE Region Using LQ-Moment

No. of Site	No. of Observation	Name of Sites	LQ-CV	LQ-Skewness	LQ-Kurtosis	D_i
1	30	Guwahati	0.1492	0.3960	0.1093	1.12
2	30	Mohanbari	0.1565	-0.0545	-0.0625	0.83
3	28	Silchar	0.1534	0.0931	0.2052	0.69
4	30	Lakhimpur	0.1518	0.2525	-0.0085	0.75
5	30	Passighat	0.1893	0.2302	0.2472	0.58
6	30	Agartala	0.2032	0.2278	-0.1384	2.22
7	30	Imphal	0.1744	0.2548	0.2233	0.14
8	30	Shillong	0.1779	0.2374	0.3275	0.62
9	26	Itanagar	0.1546	0.5586	0.5756	1.77
10	22	Dhubri	0.2042	0.0123	0.0151	1.25
11	25	Jorhat	0.1530	-0.1019	-0.1672	1.26
12	13	Lengpui	0.1339	0.2634	0.1863	0.77

Table 6: Z-Statistics Values of the Distribution

Sl.No.	Name of the Probability Distribution	Z-Statistics Values
1	GLO	2.12
2	GEV	1.50
3	LN3	1.25
4	PE3	0.91
5	GPA	0.43

Table 7: Parameters of the Best Fitting Distribution

Name of Distribution	Parameters		
	Location(ξ)	Scale(α)	Shape(k)
GPA	0.668	0.511	0.357

Table 8: Quantile Estimates by Using Best Fitting Distribution

Distribution	Quantiles									
	0.010	0.020	0.050	0.100	0.200	0.500	0.900	0.950	0.990	0.999
GPA	0.674	0.679	0.694	0.721	0.778	0.982	1.471	1.609	1.824	1.979

Table-9: RRMSE Values of Different Quantiles of PE3 Distribution and GPA Distribution for L-Moment and LQ-Moment Method Respectively

Distribution	RRMSE error									
	0.010	0.020	0.050	0.100	0.200	0.500	0.900	0.950	0.990	0.999
PE3	0.153	0.125	0.094	0.079	0.072	0.064	0.068	0.084	0.124	0.172
GPA	0.108	0.106	0.099	0.089	0.077	0.067	0.109	0.187	0.568	3.387

Table 10: RBIAS Values of Different Quantiles of PE3 Distribution and GPA Distribution for L-Moment and LQ-Moment Method Respectively

Distribution	RBIAS Error									
	0.010	0.020	0.050	0.100	0.200	0.500	0.900	0.950	0.990	0.999
PE3	-0.012	-0.005	-0.000	0.003	0.002	0.000	-0.002	-0.001	0.001	0.003
GPA	-0.006	-0.006	-0.003	-0.002	-0.001	0.004	0.028	0.058	0.188	0.676

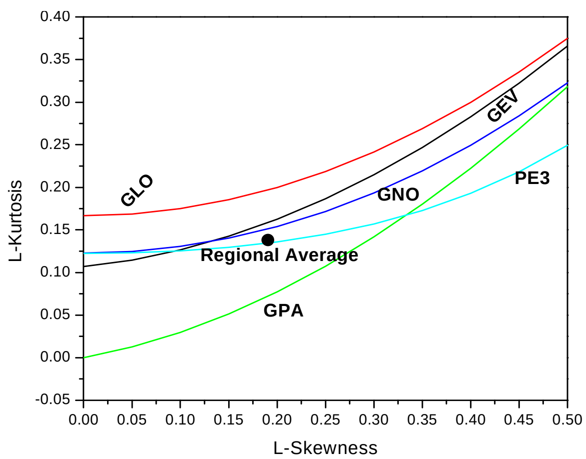

Figure 1: L-Moment Ratio Diagram for NE Region

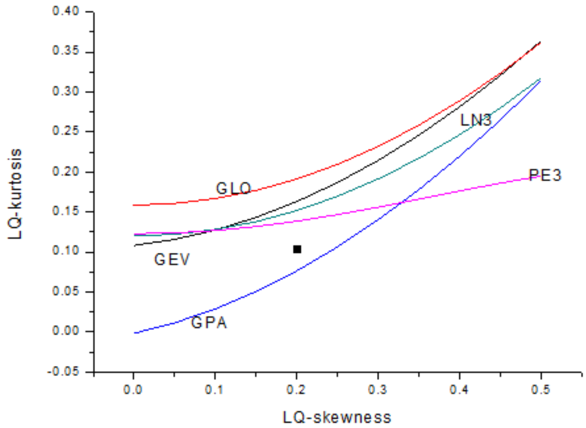

Figure 2: LQ-Moment Ratio Diagram for NE Region

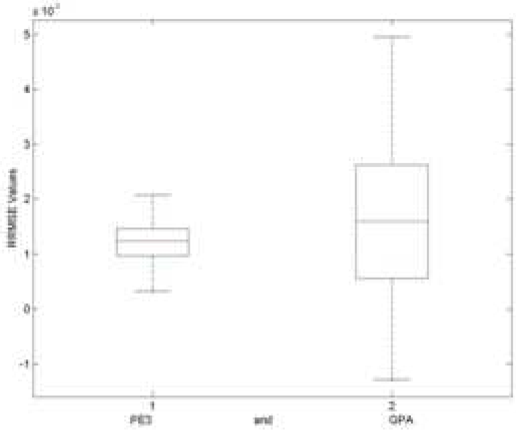

Figure 3: Boxplot of RRMSE Values

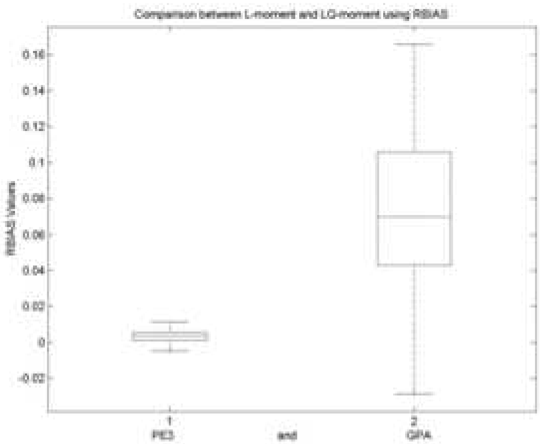

Figure 4: Boxplot of RBIAS Values